



Bridging Decision Problems:

Volume I: Framing the problem

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Bridging Problems to Models:

Volume I: Framing Decision Problems

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Contents

Preface	iii
1 Decisions, decisions, ...	1
1.1 What do we mean by a “problem”?	3
1.2 Settings for decision problems	4
1.3 The three stages of decision automation	4
1.3.1 Stage I: Framing the problem	5
1.3.1.1 Types of metrics	7
1.3.1.2 Types of decisions	8
1.3.1.3 Types of uncertainties	9
1.3.2 Stage II: Modeling	10
1.3.3 Stage III: Implementation	11
1.4 The classes of information	12
1.5 Decision making as a process	13
1.5.1 Sequential decision problems	13
1.5.2 From optimizing decisions (x) to finding functions (policies)	15
1.5.3 The information chain	16
1.6 Artificial intelligence	17
1.6.1 The seven levels of AI	17
1.6.2 Three classes of computer intelligence	23

1.6.3	Summary	25
1.7	Traditional modeling frameworks for decisions	26
1.7.1	Static, deterministic models	26
1.7.2	Sequential decision models	26
1.7.3	The most common decision problem	27
1.7.4	Static, deterministic optimization versus sequential decision problems	28
1.8	Stages of modeling	29
1.9	Types of analytics	33
1.10	Closing notes	34
1.11	Exercises	35
2	Applications	37
2.1	Getting started – solving problems	39
2.2	Modeling decision-makers	39
2.3	Framing the problem	41
2.4	Capturing interactions	41
2.4.1	Impact of decisions on metrics	42
2.4.2	Impact of uncertainty on metrics given the decision	42
2.4.3	Impact of uncertainty on what decisions we can make	43
2.4.4	Uncertainty in how the system evolves over time	44
2.4.5	Uncertainty in forecasts	44
2.4.6	Comments	44
2.5	Inventory planning	45
2.5.1	Narrative	45
2.5.2	Metrics	46
2.5.3	Decisions	47
2.5.3.1	Single inventory problem:	47
2.5.3.2	Network-level inventory decisions	48
2.5.4	Uncertainties	48
2.5.5	Interactions	50
2.6	Demand management – selling furniture	50
2.6.1	Narrative	50
2.6.2	Metrics	51
2.6.3	Decisions	52
2.6.4	Uncertainties	52

2.7	Electric Power Grid management	52
2.7.1	Narrative	53
2.7.2	Metrics	54
2.7.3	Decisions	54
2.7.4	Uncertainties	55
2.8	Hotel revenue management	56
2.8.1	Narrative	56
2.8.2	Metrics	56
2.8.3	Decisions	57
2.8.4	Uncertainties	57
2.9	Health applications	57
2.9.1	Medical decision making – managing Type 2 diabetes	57
2.9.1.1	Narrative	57
2.9.1.2	Metrics	58
2.9.1.3	Decisions	59
2.9.1.4	Uncertainties	59
2.9.2	Public health – Managing naloxone kits	60
2.9.2.1	Narrative	60
2.9.2.2	Metrics	60
2.9.2.3	Decisions	60
2.9.2.4	Uncertainties	61
2.9.3	Running clinical trials for drug testing	62
2.9.3.1	Narrative	62
2.9.3.2	Metrics	63
2.9.3.3	Decisions	63
2.9.3.4	Uncertainties	64
2.10	Running a presidential election	64
2.10.1	Narrative	64
2.10.2	Metrics	64
2.10.3	Decisions	64
2.10.4	Uncertainties	65
2.11	Truckload fleet management	65
2.11.1	Narrative	65
2.11.2	Metrics	66
2.11.3	Decisions	67
2.11.4	Uncertainties	67

2.12	Mutual fund cash management	67
2.12.1	Narrative	67
2.12.2	Metrics	69
2.12.3	Decisions	69
2.12.4	Uncertainties	69
2.13	Supply chain finance	70
2.13.1	Narrative	70
2.13.2	Metrics	71
2.13.3	Decisions	71
2.13.4	Uncertainties	72
2.14	Intelligent trial and error	72
2.14.1	Narrative	72
2.14.2	Metrics	75
2.14.3	Decisions	76
2.14.4	Uncertainties	76
2.15	Exercises	77
3	Metrics	80
3.1	Classes of metrics:	80
3.1.1	Financial	81
3.1.2	Asset productivity (facilities, equipment, people)	81
3.1.3	Product performance (machine, drug, device)	82
3.1.4	Market performance/customer satisfaction	82
3.1.5	Labor performance	83
3.1.6	Labor satisfaction	83
3.2	The metric pyramids	83
3.3	Objectives, targets and limits	84
3.4	Handling multiple objectives	85
3.5	Average performance vs. risk	85
3.6	At a point in time vs. over time	88
3.7	How people create metrics	91
3.7.1	Complex metrics	92
3.7.2	Some theories for metric formation	92
3.7.3	How the brain learns to optimize	94
3.8	Setting performance goals for others	95
3.9	Exercises	95

4	Decisions	97
4.1	Decisions and the English language	98
4.2	Identifying decisions	100
4.3	Types of decisions:	101
4.4	How decisions impact the system	102
4.5	Timing of decisions	103
4.6	Who makes decisions	104
4.7	Making decisions with computers	104
4.7.1	Policy function approximations (PFAs)	105
4.7.2	Cost function approximations (CFAs)	106
4.7.3	Value function approximations (VFAs)	107
4.7.4	Direct lookaheads (DLAs)	108
4.7.5	Hybrid policies:	110
4.7.6	Which policies are most widely used?	110
4.8	Exercises	111
5	Uncertainties	114
5.1	The 12 classes of uncertainty	115
5.2	Examples from selected applications	118
5.2.1	Cash management for a mutual fund	118
5.2.2	Finding the best diabetes treatment	118
5.2.3	Supply chain management	118
5.2.4	Allocating naloxone kits	119
5.2.5	Managing a fleet of trucks	119
5.2.6	Planning an electric power grid	120
5.3	How uncertainty affects performance	120
5.4	Different forms of uncertainty	121
5.5	Seasonality	124
5.6	Creating beliefs	125
5.7	The problem of correlations	126
5.7.1	Correlations over time	127
5.7.2	Correlations across geography	128
5.7.3	Correlations across attributes	129
5.8	Exercises	129

6 Closing notes	131
6.1 Decisions, decisions	132
6.2 Next steps	134
References	135

Bridging Problems to Models:

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ABSTRACT

There is an extensive mathematical literature on a wide range of optimization problems, from a substantial, coherent literature on deterministic optimization which has yielded a library of powerful solvers, to a highly fragmented set of communities that address optimization problems in the presence of different forms of uncertainty with very little in the way of standard solvers. Yet, surprisingly little attention has been given to the process of translating real-world decision problems, which almost universally require dealing with uncertainty, into mathematical models.

Our presentation is, necessarily, grounded in applications which span a massive class we refer to as *sequential decision problems* which evolve through a sequence of “decisions,” which we control, and “information” (more precisely “exogeneous information”) that comes from outside of our control. Sequential decision problems pretty much span any problems that involve decisions, since any static problem (where we make a set of decisions and then stop) is simply a special case.

This is the first in a series of volumes that focus on bridging the gap between real-world decision problems and models that can be implemented on the computer. This volume focuses on the first step, called “framing the problem,” which involves describing a problem in terms of terms in English that are a necessary first step to creating any model that might be translated to the computer. The first stage of the modeling process (there are three) starts by identifying performance metrics, the decisions that design and control our system, and the uncertainties that affect its performance.

A defining characteristic of this first volume is that there is *no math*. This discipline was enforced to emphasize the philosophy

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that all mathematical models start in English. However, our choice of questions is determined by what we would need to create a mathematical model. This does not mean that the lessons in this volume are only useful for building mathematical models. People make decisions every day without the use of computers. Our feeling is that the requirements of a mathematical model can help people think more clearly about a problem, even if a computer is never going to be used. We refer to this as *analytical thinking*.

This volume does not address a number of issues that are addressed in the later stages: how to make a decision, what information is needed to run our system (and how to get it), and the steps required to implement and evaluate the decisions in the field. These critically important elements of a controllable system need to wait for subsequent volumes.

Our presentation is supported by the use of as many applications as we could fit. The applications reflect the experiences of the author, with some additional problems thrown in to reflect the vast diversity of problems that involve decisions. Despite these noble intentions, we cannot come close to providing even a sample of the breadth of problems that require making decisions. Our hope is that we provide enough of a foundation to allow readers to adapt the ideas to their individual settings.

Preface

Problem solving generally starts with unstructured situations that emerge because of a desire to improve performance in some way. Problems do not always need mathematical models, but we are going to use the structure of mathematical models to guide how to think about problems. The degree to which problems need simple models on computers (think of spreadsheets), or more sophisticated models, varies from one problem to the next. Ultimately, we are going to use a very general modeling framework to guide how we think about problems.

For most of my career, I approached problems as shown in Figure 1. I would start with a physical problem, translate it to a mathematical model, and then implement it on the computer. The problem with this approach is that I would start with a particular mathematical modeling style, and then collect the data needed to fill in my preconceived modeling framework.

I have come to learn that this approach breaks down in the context of the richer problems that arise in practice. The limitations of my initial approach would reveal itself as it became clear that the model was not solving the real problem. This would start an iterative process that I called “From the lab, to the field, and back.”

As I tackled an ever-evolving range of problems, I managed to develop a more flexible modeling framework that captures a much richer set of problems that I now call sequential decision problems. This shifted the emphasis from modeling a particular problem to understanding the problem before the modeling process was started.

The process of understanding complex problems such as those that arise throughout business, but also health (especially public health), energy and some financial applications, has long been recognized by consultants who

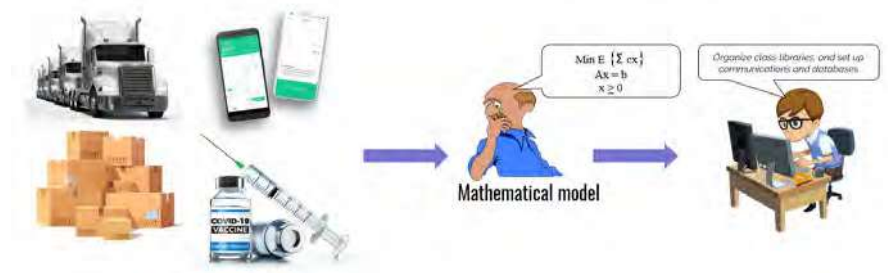


Figure 1: The bridge between the real world and the computer is a mathematical model.

approach the problems under the umbrella of terms like “decision analysis” where they recognize a step they call “framing the problem.” This may be done without regard to the use of any particular tool.

Professional optimization specialists, on the other hand, approach the process as I would, looking to fill in the blanks for a well-defined modeling framework that would fulfill the requirements of optimization software for solving any of a number of (typically deterministic) optimization problems. The problem is that the modeling process is performed within the limitations of the optimization software. The most prominent limitation has been the handling of uncertainty which is pervasive in real applications.

This book is motivated by the development of a very general modeling strategy we call the “universal modeling framework” which was designed to represent any sequential decision problem. The framework covers a virtually unlimited range of problems (static problems are just one special case), justifying the use of the adjective “universal.” The defining philosophy of this framework is given by:

Model first, then solve.

This means modeling the problem before deciding how to “solve” it (by this I mean, how to make the decisions that allow us to achieve an improved solution).

Sequential decision problems have been approached in the research literature by over a dozen distinct communities, using eight different notational systems, a number of modeling frameworks and a range of tools motivated by applications from different problem settings. Books representing each of the different communities are depicted in figure 2. Every one of these books assumes that we are solving a problem with a well-defined objective function,

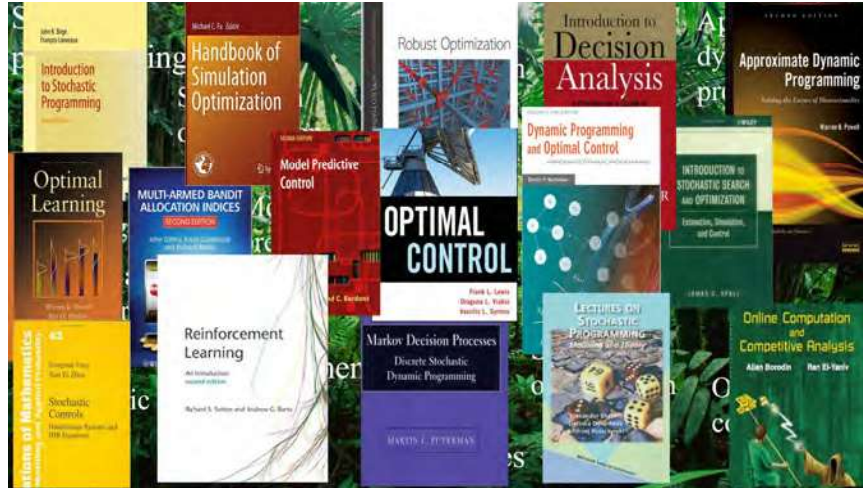


Figure 2: The “jungle of stochastic optimization” capturing the contributions by many communities to the problem of making decisions under uncertainty.

pre-defined decisions and a clear model of the uncertainties.

This volume makes none of these assumptions. Most important, we do not even start with a mathematical model. Instead, we develop a process that we refer to as “framing the problem” which consists of asking questions where the answers form of the foundation of the mathematical model in the universal modeling framework. We design a framing process that, as with other efforts at framing, starts in English. But rather than the very general language of consultants wading through complex business problems, our framing process is guided by the universal modeling framework which provides a direct path to a computer model.

Our universal modeling framework has three critical features:

- 1) It is completely general, in that it can represent *any* sequential decision problem (static problems are simply a special case).
- 2) The framework does not make any preconceived assumptions about how decisions will be made.
- 3) Decisions are made with methods called “policies.” We describe four classes of policies which, combined with hybrids of two or more classes, include *any* method for making decisions, including whatever method is being used in practice.

The modeling framework is so general that it allows us to approach any problem that involves making decisions. Instead of solving well-defined math problems

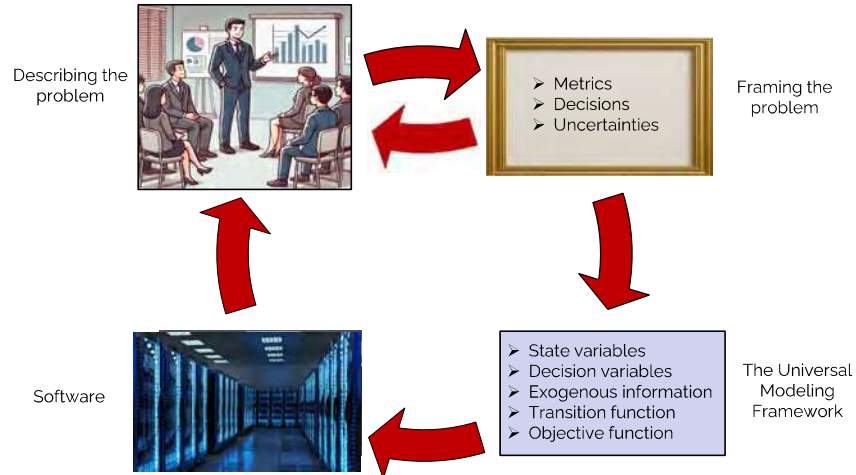


Figure 3: The modeling cycle: From problem, to framing, to model, to software, back to problem.

that are posed in the books in figure 2, we need a more general vocabulary that can capture the richness of problems in this class.

To accomplish this, we are replacing figure 1, which steps from problem to model to software, to the one depicted in figure 3, where we start with a general statement of the original problem described in the language of domain experts. We then transition to the framing step, which involves answering a series of questions in English, but which produces answers that translate directly to the universal modeling framework, without any preconceived notion of how we are going to solve the problem.

There are three stages in the process of designing and implementing models, but this volume will focus on the first one that starts by looking for answers to three questions:

- 1) What are the performance metrics?
- 2) What types of decisions are being made (and who makes them)?
- 3) What are the forms of uncertainty that affect the performance of the system?

By insisting that this entire discussion be made in English, we avoid the subtle assumptions that insert themselves when using a mathematical model. However, in Volume II, we will show how to convert the answers to these questions into mathematics. Without this step, the volume would devolve into

the discussions that can be found in any of a number of business books which may be entertaining to executives, but would never be implementable on a computer.

Acknowledgments

This volume has grown out of a lifetime of research developing models and algorithms for a wide range of complex problems. This work could not have been done without the help of over 60 graduate students, post-docs and my valuable research staff. This volume has also been informed by several of the over 200 senior theses that I have supervised at Princeton University. This research laid the foundation for the universal modeling framework for modeling any sequential decision problem, and the four classes of policies for making decisions. All of this work is contained in my 2022 volume Powell, 2022.

The idea of starting with the step of answering three questions (about metrics, decisions and uncertainties) has its origins in a talk I gave at a workshop for Professor Steven Platt at the University of Loyola in 2022, where I presented my universal modeling framework to a group of (very nonanalytical) business executives.

It was not until 2024 where, working with the Department of Supply Chain Management at Rutgers University, I realized that this department (also with nonanalytical students) could provide the incredibly valuable role of framing the problem which is a critical bridge between real applications and my modeling framework. I am grateful to Professor Lian Qi, chair of the Rutgers SCM department, for giving me the opportunity to interact with his students and faculty.

The last step was meeting Adam DeJans at Toyota (through LinkedIn) who demonstrated a real command of my universal modeling framework. From his position at the world's largest automotive manufacturer, Adam recognized its usefulness in the diverse and complex problems faced by Toyota. He invited me to give a talk at Toyota, and it was the preparation for this talk, to a broad audience including top-level executives, that filled in some of the remaining pieces that are reflected in this book.

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