Reinforcement Learning and Stochastic Optimization
A Unified Framework for Sequential Decisions

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A Unified Framework for Sequential Decisions

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Contents

Preface *xxv*
Acknowledgments *xxxi*

Part I – Introduction *1*

1 Sequential Decision Problems *3*
1.1 The Audience *7*
1.2 The Communities of Sequential Decision Problems *8*
1.3 Our Universal Modeling Framework *10*
1.4 Designing Policies for Sequential Decision Problems *15*
1.4.1 Policy Search *15*
1.4.2 Policies Based on Lookahead Approximations *17*
1.4.3 Mixing and Matching *18*
1.4.4 Optimality of the Four Classes *19*
1.4.5 Pulling it All Together *19*
1.5 Learning *20*
1.6 Themes *21*
1.6.1 Blending Learning and Optimization *21*
1.6.2 Bridging Machine Learning to Sequential Decisions *21*
1.6.3 From Deterministic to Stochastic Optimization *23*
1.6.4 From Single to Multiple Agents *26*
1.7 Our Modeling Approach *27*
1.8 How to Read this Book *27*
1.8.1 Organization of Topics *28*
1.8.2 How to Read Each Chapter *31*
1.8.3 Organization of Exercises *32*

1.9 Bibliographic Notes *33*
Exercises *34*
Bibliography *38*

2 Canonical Problems and Applications *39*
2.1 Canonical Problems *39*
2.1.1 Stochastic Search – Derivative-based and Derivative-free *40*
2.1.1.1 Derivative-based Stochastic Search *42*
2.1.1.2 Derivative-free Stochastic Search *43*
2.1.2 Decision Trees *44*
2.1.3 Markov Decision Processes *45*
2.1.4 Optimal Control *47*
2.1.5 Approximate Dynamic Programming *50*
2.1.6 Reinforcement Learning *50*
2.1.7 Optimal Stopping *54*
2.1.8 Stochastic Programming *56*
2.1.9 The Multiarmed Bandit Problem *57*
2.1.10 Simulation Optimization *60*
2.1.11 Active Learning *61*
2.1.12 Chance-constrained Programming *61*
2.1.13 Model Predictive Control *62*
2.1.14 Robust Optimization *63*
2.2 A Universal Modeling Framework for Sequential Decision Problems *64*
2.2.1 Our Universal Model for Sequential Decision Problems *65*
2.2.2 A Compact Modeling Presentation *68*
2.2.3 MDP/RL vs. Optimal Control Modeling Frameworks *68*
2.3 Applications *69*
2.3.1 The Newsvendor Problems *70*
2.3.1.1 Basic Newsvendor – Final Reward *70*
2.3.1.2 Basic Newsvendor – Cumulative Reward *71*
2.3.1.3 Contextual Newsvendor *71*
2.3.1.4 Multidimensional Newsvendor Problems *72*
2.3.2 Inventory/Storage Problems *73*
2.3.2.1 Inventory Without Lags *73*
2.3.2.2 Inventory Planning with Forecasts *75*
2.3.2.3 Lagged Decisions *75*
2.3.3 Shortest Path Problems *76*
2.3.3.1 A Deterministic Shortest Path Problem *76*
2.3.3.2 A Stochastic Shortest Path Problem *77*
2.3.3.3 A Dynamic Shortest Path Problem *78*

2.3.3.4 A Robust Shortest Path Problem *78*
2.3.4 Some Fleet Management Problems *78*
2.3.4.1 The Nomadic Trucker *79*
2.3.4.2 From One Driver to a Fleet *80*
2.3.5 Pricing *80*
2.3.6 Medical Decision Making *81*
2.3.7 Scientific Exploration *82*
2.3.8 Machine Learning vs. Sequential Decision Problems *83*
2.4 Bibliographic Notes *85*
Exercises *90*
Bibliography *93*

3 Online Learning *101*
3.1 Machine Learning for Sequential Decisions *102*
3.1.1 Observations and Data in Stochastic Optimization *102*
3.1.2 Indexing Input x^n and Response y^{n+1} *103*
3.1.3 Functions We are Learning *103*
3.1.4 Sequential Learning: From Very Little Data to ... More Data *105*
3.1.5 Approximation Strategies *106*
3.1.6 From Data Analytics to Decision Analytics *108*
3.1.7 Batch vs. Online Learning *109*
3.2 Adaptive Learning Using Exponential Smoothing *110*
3.3 Lookup Tables with Frequentist Updating *111*
3.4 Lookup Tables with Bayesian Updating *112*
3.4.1 The Updating Equations for Independent Beliefs *113*
3.4.2 Updating for Correlated Beliefs *113*
3.4.3 Gaussian Process Regression *117*
3.5 Computing Bias and Variance* *118*
3.6 Lookup Tables and Aggregation* *121*
3.6.1 Hierarchical Aggregation *121*
3.6.2 Estimates of Different Levels of Aggregation *125*
3.6.3 Combining Multiple Levels of Aggregation *129*
3.7 Linear Parametric Models *131*
3.7.1 Linear Regression Review *132*
3.7.2 Sparse Additive Models and Lasso *134*
3.8 Recursive Least Squares for Linear Models *136*
3.8.1 Recursive Least Squares for Stationary Data *136*
3.8.2 Recursive Least Squares for Nonstationary Data* *138*
3.8.3 Recursive Estimation Using Multiple Observations* *139*
3.9 Nonlinear Parametric Models *140*

3.9.1 Maximum Likelihood Estimation *141*
3.9.2 Sampled Belief Models *141*
3.9.3 Neural Networks – Parametric* *143*
3.9.4 Limitations of Neural Networks *148*
3.10 Nonparametric Models* *149*
3.10.1 K-Nearest Neighbor *150*
3.10.2 Kernel Regression *151*
3.10.3 Local Polynomial Regression *153*
3.10.4 Deep Neural Networks *154*
3.10.5 Support Vector Machines *155*
3.10.6 Indexed Functions, Tree Structures, and Clustering *156*
3.10.7 Comments on Nonparametric Models *157*
3.11 Nonstationary Learning* *159*
3.11.1 Nonstationary Learning I – Martingale Truth *159*
3.11.2 Nonstationary Learning II – Transient Truth *160*
3.11.3 Learning Processes *161*
3.12 The Curse of Dimensionality *162*
3.13 Designing Approximation Architectures in Adaptive Learning *165*
3.14 Why Does It Work?** *166*
3.14.1 Derivation of the Recursive Estimation Equations *166*
3.14.2 The Sherman-Morrison Updating Formula *168*
3.14.3 Correlations in Hierarchical Estimation *169*
3.14.4 Proof of Proposition 3.14.1 *172*
3.15 Bibliographic Notes *174*
Exercises *176*
Bibliography *180*

4 Introduction to Stochastic Search *183*
4.1 Illustrations of the Basic Stochastic Optimization Problem *185*
4.2 Deterministic Methods *188*
4.2.1 A "Stochastic" Shortest Path Problem *189*
4.2.2 A Newsvendor Problem with Known Distribution *189*
4.2.3 Chance-Constrained Optimization *190*
4.2.4 Optimal Control *191*
4.2.5 Discrete Markov Decision Processes *192*
4.2.6 Remarks *192*
4.3 Sampled Models *193*
4.3.1 Formulating a Sampled Model *194*
4.3.1.1 A Sampled Stochastic Linear Program *194*

4.3.1.2 Sampled Chance-Constrained Models *195*
4.3.1.3 Sampled Parametric Models *196*
4.3.2 Convergence *197*
4.3.3 Creating a Sampled Model *199*
4.3.4 Decomposition Strategies* *200*
4.4 Adaptive Learning Algorithms *202*
4.4.1 Modeling Adaptive Learning Problems *202*
4.4.2 Online vs. Offline Applications *204*
4.4.2.1 Machine Learning *204*
4.4.2.2 Optimization *205*
4.4.3 Objective Functions for Learning *205*
4.4.4 Designing Policies *209*
4.5 Closing Remarks *210*
4.6 Bibliographic Notes *210*
Exercises *212*
Bibliography *218*

Part II – Stochastic Search *221*

5 Derivative-Based Stochastic Search *223*
5.1 Some Sample Applications *225*
5.2 Modeling Uncertainty *228*
5.2.1 Training Uncertainty $W^1, \ldots, W^N$ *228*
5.2.2 Model Uncertainty S^0 *229*
5.2.3 Testing Uncertainty *230*
5.2.4 Policy Evaluation *231*
5.2.5 Closing Notes *231*
5.3 Stochastic Gradient Methods *231*
5.3.1 A Stochastic Gradient Algorithm *232*
5.3.2 Introduction to Stepsizes *233*
5.3.3 Evaluating a Stochastic Gradient Algorithm *235*
5.3.4 A Note on Notation *236*
5.4 Styles of Gradients *237*
5.4.1 Gradient Smoothing *237*
5.4.2 Second-Order Methods *237*
5.4.3 Finite Differences *238*
5.4.4 SPSA *240*
5.4.5 Constrained Problems *242*
5.5 Parameter Optimization for Neural Networks* *242*
5.5.1 Computing the Gradient *244*

5.5.2 The Stochastic Gradient Algorithm *246*
5.6 Stochastic Gradient Algorithm as a Sequential Decision Problem *247*
5.7 Empirical Issues *248*
5.8 Transient Problems* *249*
5.9 Theoretical Performance* *250*
5.10 Why Does it Work? *250*
5.10.1 Some Probabilistic Preliminaries *251*
5.10.2 An Older Proof* *252*
5.10.3 A More Modern Proof** *256*
5.11 Bibliographic Notes *263*
Exercises *264*
Bibliography *270*

6 Stepsize Policies *273*
6.1 Deterministic Stepsize Policies *276*
6.1.1 Properties for Convergence *276*
6.1.2 A Collection of Deterministic Policies *278*
6.1.2.1 Constant Stepsizes *278*
6.1.2.2 Generalized Harmonic Stepsizes *279*
6.1.2.3 Polynomial Learning Rates *280*
6.1.2.4 McClain's Formula *280*
6.1.2.5 Search-then-Converge Learning Policy *281*
6.2 Adaptive Stepsize Policies *282*
6.2.1 The Case for Adaptive Stepsizes *283*
6.2.2 Convergence Conditions *283*
6.2.3 A Collection of Stochastic Policies *284*
6.2.3.1 Kesten's Rule *285*
6.2.3.2 Trigg's Formula *286*
6.2.3.3 Stochastic Gradient Adaptive Stepsize Rule *286*
6.2.3.4 ADAM *287*
6.2.3.5 AdaGrad *287*
6.2.3.6 RMSProp *288*
6.2.4 Experimental Notes *289*
6.3 Optimal Stepsize Policies* *289*
6.3.1 Optimal Stepsizes for Stationary Data *291*
6.3.2 Optimal Stepsizes for Nonstationary Data – I *293*
6.3.3 Optimal Stepsizes for Nonstationary Data – II *294*
6.4 Optimal Stepsizes for Approximate Value Iteration* *297*
6.5 Convergence *300*

6.6 Guidelines for Choosing Stepsize Formulas *301*
6.7 Why Does it Work* *303*
6.7.1 Proof of BAKF Stepsize *303*
6.8 Bibliographic Notes *306*
Exercises *307*
Bibliography *314*

7 Derivative-Free Stochastic Search *317*
7.1 Overview of Derivative-free Stochastic Search *319*
7.1.1 Applications and Time Scales *319*
7.1.2 The Communities of Derivative-free Stochastic Search *321*
7.1.3 The Multiarmed Bandit Story *321*
7.1.4 From Passive Learning to Active Learning to Bandit Problems *323*
7.2 Modeling Derivative-free Stochastic Search *325*
7.2.1 The Universal Model *325*
7.2.2 Illustration: Optimizing a Manufacturing Process *328*
7.2.3 Major Problem Plasses *329*
7.3 Designing Policies *330*
7.4 Policy Function Approximations *333*
7.5 Cost Function Approximations *335*
7.6 VFA-based Policies *338*
7.6.1 An Optimal Policy *338*
7.6.2 Beta-Bernoulli Belief Model *340*
7.6.3 Backward Approximate Dynamic Programming *342*
7.6.4 Gittins Indices for Learning in Steady State *343*
7.7 Direct Lookahead Policies *348*
7.7.1 When do we Need Lookahead Policies? *349*
7.7.2 Single Period Lookahead Policies *350*
7.7.3 Restricted Multiperiod Lookahead *353*
7.7.4 Multiperiod Deterministic Lookahead *355*
7.7.5 Multiperiod Stochastic Lookahead Policies *357*
7.7.6 Hybrid Direct Lookahead *360*
7.8 The Knowledge Gradient (Continued)* *362*
7.8.1 The Belief Model *363*
7.8.2 The Knowledge Gradient for Maximizing Final Reward *364*
7.8.3 The Knowledge Gradient for Maximizing Cumulative Reward *369*
7.8.4 The Knowledge Gradient for Sampled Belief Model* *370*
7.8.5 Knowledge Gradient for Correlated Beliefs *375*
7.9 Learning in Batches *380*
7.10 Simulation Optimization* *382*
7.10.1 An Indifference Zone Algorithm *383*

7.10.2 Optimal Computing Budget Allocation *383*
7.11 Evaluating Policies *385*
7.11.1 Alternative Performance Metrics* *386*
7.11.2 Perspectives of Optimality* *392*
7.12 Designing Policies *394*
7.12.1 Characteristics of a Policy *395*
7.12.2 The Effect of Scaling *396*
7.12.3 Tuning *398*
7.13 Extensions* *398*
7.13.1 Learning in Nonstationary Settings *399*
7.13.2 Strategies for Designing Time-dependent Policies *400*
7.13.3 A Transient Learning Model *401*
7.13.4 The Knowledge Gradient for Transient Problems *402*
7.13.5 Learning with Large or Continuous Choice Sets *403*
7.13.6 Learning with Exogenous State Information – the Contextual Bandit Problem *405*
7.13.7 State-dependent vs. State-independent Problems *408*
7.14 Bibliographic Notes *409*
Exercises *412*
Bibliography *424*

Part III – State-dependent Problems *429*

8 State-dependent Problems *431*
8.1 Graph Problems *433*
8.1.1 A Stochastic Shortest Path Problem *433*
8.1.2 The Nomadic Trucker *434*
8.1.3 The Transformer Replacement Problem *435*
8.1.4 Asset Valuation *437*
8.2 Inventory Problems *439*
8.2.1 A Basic Inventory Problem *439*
8.2.2 The Inventory Problem – II *440*
8.2.3 The Lagged Asset Acquisition Problem *443*
8.2.4 The Batch Replenishment Problem *444*
8.3 Complex Resource Allocation Problems *446*
8.3.1 The Dynamic Assignment Problem *447*
8.3.2 The Blood Management Problem *450*
8.4 State-dependent Learning Problems *456*
8.4.1 Medical Decision Making *457*
8.4.2 Laboratory Experimentation *458*

8.4.3 Bidding for Ad-clicks *459*
8.4.4 An Information-collecting Shortest Path Problem *459*
8.5 A Sequence of Problem Classes *460*
8.6 Bibliographic Notes *461*
Exercises *462*
Bibliography *466*

9 Modeling Sequential Decision Problems *467*
9.1 A Simple Modeling Illustration *471*
9.2 Notational Style *476*
9.3 Modeling Time *478*
9.4 The States of Our System *481*
9.4.1 Defining the State Variable *481*
9.4.2 The Three States of Our System *485*
9.4.3 Initial State S_0 vs. Subsequent States $S_t,\ t > 0$ *488*
9.4.4 Lagged State Variables* *490*
9.4.5 The Post-decision State Variable* *490*
9.4.6 A Shortest Path Illustration *493*
9.4.7 Belief States* *495*
9.4.8 Latent Variables* *496*
9.4.9 Rolling Forecasts* *497*
9.4.10 Flat vs. Factored State Representations* *498*
9.4.11 A Programmer's Perspective of State Variables *499*
9.5 Modeling Decisions *500*
9.5.1 Types of Decisions *502*
9.5.2 Initial Decision x_0 vs. Subsequent Decisions $x_t,\ t > 0$ *502*
9.5.3 Strategic, Tactical, and Execution Decisions *503*
9.5.4 Constraints *504*
9.5.5 Introducing Policies *505*
9.6 The Exogenous Information Process *506*
9.6.1 Basic Notation for Information Processes *506*
9.6.2 Outcomes and Scenarios *509*
9.6.3 Lagged Information Processes* *510*
9.6.4 Models of Information Processes* *511*
9.6.5 Supervisory Processes* *514*
9.7 The Transition Function *515*
9.7.1 A General Model *515*
9.7.2 Model-free Dynamic Programming *516*
9.7.3 Exogenous Transitions *518*
9.8 The Objective Function *518*
9.8.1 The Performance Metric *518*

9.8.2 Optimizing the Policy *519*
9.8.3 Dependence of Optimal Policy on S_0 *520*
9.8.4 State-dependent Variations *520*
9.8.5 Uncertainty Operators *523*
9.9 Illustration: An Energy Storage Model *523*
9.9.1 With a Time-series Price Model *525*
9.9.2 With Passive Learning *525*
9.9.3 With Active Learning *526*
9.9.4 With Rolling Forecasts *526*
9.10 Base Models and Lookahead Models *528*
9.11 A Classification of Problems* *529*
9.12 Policy Evaluation* *532*
9.13 Advanced Probabilistic Modeling Concepts** *534*
9.13.1 A Measure-theoretic View of Information** *535*
9.13.2 Policies and Measurability *538*
9.14 Looking Forward *540*
9.15 Bibliographic Notes *542*
Exercises *544*
Bibliography *557*

10 Uncertainty Modeling *559*
10.1 Sources of Uncertainty *560*
10.1.1 Observational Errors *562*
10.1.2 Exogenous Uncertainty *564*
10.1.3 Prognostic Uncertainty *564*
10.1.4 Inferential (or Diagnostic) Uncertainty *567*
10.1.5 Experimental Variability *568*
10.1.6 Model Uncertainty *569*
10.1.7 Transitional Uncertainty *571*
10.1.8 Control/implementation Uncertainty *571*
10.1.9 Communication Errors and Biases *572*
10.1.10 Algorithmic Instability *573*
10.1.11 Goal Uncertainty *574*
10.1.12 Political/regulatory Uncertainty *574*
10.1.13 Discussion *574*
10.2 A Modeling Case Study: The COVID Pandemic *575*
10.3 Stochastic Modeling *575*
10.3.1 Sampling Exogenous Information *575*
10.3.2 Types of Distributions *577*
10.3.3 Modeling Sample Paths *578*
10.3.4 State-action-dependent Processes *579*

10.3.5 Modeling Correlations *581*
10.4 Monte Carlo Simulation *581*
10.4.1 Generating Uniform [0, 1] Random Variables *582*
10.4.2 Uniform and Normal Random Variable *583*
10.4.3 Generating Random Variables from Inverse Cumulative Distributions *585*
10.4.4 Inverse Cumulative From Quantile Distributions *586*
10.4.5 Distributions with Uncertain Parameters *587*
10.5 Case Study: Modeling Electricity Prices *589*
10.5.1 Mean Reversion *590*
10.5.2 Jump-diffusion Models *590*
10.5.3 Quantile Distributions *591*
10.5.4 Regime Shifting *592*
10.5.5 Crossing Times *593*
10.6 Sampling vs. Sampled Models *595*
10.6.1 Iterative Sampling: A Stochastic Gradient Algorithm *595*
10.6.2 Static Sampling: Solving a Sampled Model *595*
10.6.3 Sampled Representation with Bayesian Updating *596*
10.7 Closing Notes *597*
10.8 Bibliographic Notes *597*
Exercises *598*
Bibliography *601*

11 Designing Policies *603*
11.1 From Optimization to Machine Learning to Sequential Decision Problems *605*
11.2 The Classes of Policies *606*
11.3 Policy Function Approximations *610*
11.4 Cost Function Approximations *613*
11.5 Value Function Approximations *614*
11.6 Direct Lookahead Approximations *616*
11.6.1 The Basic Idea *616*
11.6.2 Modeling the Lookahead Problem *619*
11.6.3 The Policy-Within-a-Policy *620*
11.7 Hybrid Strategies *620*
11.7.1 Cost Function Approximation with Policy Function Approximations *621*
11.7.2 Lookahead Policies with Value Function Approximations *622*
11.7.3 Lookahead Policies with Cost Function Approximations *623*
11.7.4 Tree Search with Rollout Heuristic and a Lookup Table Policy *623*

11.7.5 Value Function Approximation with Policy Function Approximation *624*
11.7.6 Fitting Value Functions Using ADP and Policy Search *624*
11.8 Randomized Policies *626*
11.9 Illustration: An Energy Storage Model Revisited *627*
11.9.1 Policy Function Approximation *628*
11.9.2 Cost Function Approximation *628*
11.9.3 Value Function Approximation *628*
11.9.4 Deterministic Lookahead *629*
11.9.5 Hybrid Lookahead-Cost Function Approximation *629*
11.9.6 Experimental Testing *629*
11.10 Choosing the Policy Class *631*
11.10.1 The Policy Classes *631*
11.10.2 Policy Complexity-Computational Tradeoffs *636*
11.10.3 Screening Questions *638*
11.11 Policy Evaluation *641*
11.12 Parameter Tuning *642*
11.12.1 The Soft Issues *644*
11.12.2 Searching Across Policy Classes *645*
11.13 Bibliographic Notes *646*
Exercises *646*
Bibliography *651*

Part IV – Policy Search *653*

12 Policy Function Approximations and Policy Search *655*
12.1 Policy Search as a Sequential Decision Problem *657*
12.2 Classes of Policy Function Approximations *658*
12.2.1 Lookup Table Policies *659*
12.2.2 Boltzmann Policies for Discrete Actions *659*
12.2.3 Linear Decision Rules *660*
12.2.4 Monotone Policies *661*
12.2.5 Nonlinear Policies *662*
12.2.6 Nonparametric/Locally Linear Policies *663*
12.2.7 Contextual Policies *665*
12.3 Problem Characteristics *665*
12.4 Flavors of Policy Search *666*
12.5 Policy Search with Numerical Derivatives *669*
12.6 Derivative-Free Methods for Policy Search *670*
12.6.1 Belief Models *671*

12.6.2 Learning Through Perturbed PFAs *672*
12.6.3 Learning CFAs *675*
12.6.4 DLA Using the Knowledge Gradient *677*
12.6.5 Comments *677*
12.7 Exact Derivatives for Continuous Sequential Problems* *677*
12.8 Exact Derivatives for Discrete Dynamic Programs** *680*
12.8.1 A Stochastic Policy *681*
12.8.2 The Objective Function *683*
12.8.3 The Policy Gradient Theorem *683*
12.8.4 Computing the Policy Gradient *684*
12.9 Supervised Learning *686*
12.10 Why Does it Work? *687*
12.10.1 Derivation of the Policy Gradient Theorem *687*
12.11 Bibliographic Notes *690*
Exercises *691*
Bibliography *698*

13 Cost Function Approximations *701*
13.1 General Formulation for Parametric CFA *703*
13.2 Objective-Modified CFAs *704*
13.2.1 Linear Cost Function Correction *705*
13.2.2 CFAs for Dynamic Assignment Problems *705*
13.2.3 Dynamic Shortest Paths *707*
13.2.4 Dynamic Trading Policy *711*
13.2.5 Discussion *713*
13.3 Constraint-Modified CFAs *714*
13.3.1 General Formulation of Constraint-Modified CFAs *715*
13.3.2 A Blood Management Problem *715*
13.3.3 An Energy Storage Example with Rolling Forecasts *717*
13.4 Bibliographic Notes *725*
Exercises *726*
Bibliography *729*

Part V – Lookahead Policies *731*

14 Exact Dynamic Programming *737*
14.1 Discrete Dynamic Programming *738*
14.2 The Optimality Equations *740*

14.2.1 Bellman's Equations *741*
14.2.2 Computing the Transition Matrix *745*
14.2.3 Random Contributions *746*
14.2.4 Bellman's Equation Using Operator Notation* *746*
14.3 Finite Horizon Problems *747*
14.4 Continuous Problems with Exact Solutions *750*
14.4.1 The Gambling Problem *751*
14.4.2 The Continuous Budgeting Problem *752*
14.5 Infinite Horizon Problems* *755*
14.6 Value Iteration for Infinite Horizon Problems* *757*
14.6.1 A Gauss-Seidel Variation *758*
14.6.2 Relative Value Iteration *758*
14.6.3 Bounds and Rates of Convergence *760*
14.7 Policy Iteration for Infinite Horizon Problems* *762*
14.8 Hybrid Value-Policy Iteration* *764*
14.9 Average Reward Dynamic Programming* *765*
14.10 The Linear Programming Method for Dynamic Programs** *766*
14.11 Linear Quadratic Regulation *767*
14.12 Why Does it Work?** *770*
14.12.1 The Optimality Equations *770*
14.12.2 Convergence of Value Iteration *774*
14.12.3 Monotonicity of Value Iteration *778*
14.12.4 Bounding the Error from Value Iteration *780*
14.12.5 Randomized Policies *781*
14.13 Bibliographic Notes *783*
Exercises *783*
Bibliography *793*

15 Backward Approximate Dynamic Programming *795*
15.1 Backward Approximate Dynamic Programming for Finite Horizon Problems *797*
15.1.1 Some Preliminaries *797*
15.1.2 Backward ADP Using Lookup Tables *799*
15.1.3 Backward ADP Algorithm with Continuous Approximations *801*
15.2 Fitted Value Iteration for Infinite Horizon Problems *804*
15.3 Value Function Approximation Strategies *805*
15.3.1 Linear Models *806*
15.3.2 Monotone Functions *807*
15.3.3 Other Approximation Models *809*

15.4 Computational Observations *809*
15.4.1 Experimental Benchmarking of Backward ADP *810*
15.4.2 Computational Notes *815*
15.5 Bibliographic Notes *816*
Exercises *816*
Bibliography *821*

16 Forward ADP I: The Value of a Policy *823*
16.1 Sampling the Value of a Policy *824*
16.1.1 Direct Policy Evaluation for Finite Horizon Problems *824*
16.1.2 Policy Evaluation for Infinite Horizon Problems *826*
16.1.3 Temporal Difference Updates *828*
16.1.4 TD(λ) *829*
16.1.5 TD(0) and Approximate Value Iteration *830*
16.1.6 TD Learning for Infinite Horizon Problems *832*
16.2 Stochastic Approximation Methods *835*
16.3 Bellman's Equation Using a Linear Model* *837*
16.3.1 A Matrix-based Derivation** *837*
16.3.2 A Simulation-based Implementation *840*
16.3.3 Least Squares Temporal Differences (LSTD) *840*
16.3.4 Least Squares Policy Evaluation (LSPE) *841*
16.4 Analysis of TD(0), LSTD, and LSPE Using a Single State* *842*
16.4.1 Recursive Least Squares and TD(0) *842*
16.4.2 LSPE *844*
16.4.3 LSTD *844*
16.4.4 Discussion *844*
16.5 Gradient-based Methods for Approximate Value Iteration* *845*
16.5.1 Approximate Value Iteration with Linear Models** *845*
16.5.2 A Geometric View of Linear Models* *850*
16.6 Value Function Approximations Based on Bayesian Learning* *852*
16.6.1 Minimizing Bias for Infinite Horizon Problems *852*
16.6.2 Lookup Tables with Correlated Beliefs *853*
16.6.3 Parametric Models *854*
16.6.4 Creating the Prior *855*
16.7 Learning Algorithms and Atepsizes *855*
16.7.1 Least Squares Temporal Differences *856*
16.7.2 Least Squares Policy Evaluation *857*
16.7.3 Recursive Least Squares *857*

16.7.4 Bounding $1/n$ Convergence for Approximate value Iteration *859*
16.7.5 Discussion *860*
16.8 Bibliographic Notes *860*
Exercises *861*
Bibliography *864*

17 Forward ADP II: Policy Optimization *867*
17.1 Overview of Algorithmic Strategies *869*
17.2 Approximate Value Iteration and $\boldsymbol{Q}$-Learning Using Lookup Tables *871*
17.2.1 Value Iteration Using a Pre-Decision State Variable *872*
17.2.2 $\boldsymbol{Q}$-Learning *873*
17.2.3 Value Iteration Using a Post-Decision State Variable *875*
17.2.4 Value Iteration Using a Backward Pass *877*
17.3 Styles of Learning *881*
17.3.1 Offline Learning *882*
17.3.2 From Offline to Online *883*
17.3.3 Evaluating Offline and Online Learning Policies *885*
17.3.4 Lookahead Policies *885*
17.4 Approximate Value Iteration Using Linear Models *886*
17.5 On-policy vs. off-policy learning and the exploration–exploitation problem *888*
17.5.1 Terminology *889*
17.5.2 Learning with Lookup Tables *890*
17.5.3 Learning with Generalized Belief Models *891*
17.6 Applications *894*
17.6.1 Pricing an American Option *894*
17.6.2 Playing "Lose Tic-Tac-Toe" *898*
17.6.3 Approximate Dynamic Programming for Deterministic Problems *900*
17.7 Approximate Policy Iteration *900*
17.7.1 Finite Horizon Problems Using Lookup Tables *901*
17.7.2 Finite Horizon Problems Using Linear Models *903*
17.7.3 LSTD for Infinite Horizon Problems Using Linear Models *903*
17.8 The Actor–Critic Paradigm *907*
17.9 Statistical Bias in the Max Operator* *909*
17.10 The Linear Programming Method Using Linear Models* *912*
17.11 Finite Horizon Approximations for Steady-State Applications *915*

17.12 Bibliographic Notes *917*
Exercises *918*
Bibliography *924*

18 Forward ADP III: Convex Resource Allocation Problems *927*
18.1 Resource Allocation Problems *930*
18.1.1 The Newsvendor Problem *930*
18.1.2 Two-Stage Resource Allocation Problems *931*
18.1.3 A General Multiperiod Resource Allocation Model* *934*
18.2 Values Versus Marginal Values *937*
18.3 Piecewise Linear Approximations for Scalar Functions *938*
18.3.1 The Leveling Algorithm *939*
18.3.2 The CAVE Algorithm *941*
18.4 Regression Methods *941*
18.5 Separable Piecewise Linear Approximations *944*
18.6 Benders Decomposition for Nonseparable Approximations** *946*
18.6.1 Benders' Decomposition for Two-Stage Problems *947*
18.6.2 Asymptotic Analysis of Benders with Regularization** *952*
18.6.3 Benders with Regularization *956*
18.7 Linear Approximations for High-Dimensional Applications *956*
18.8 Resource Allocation with Exogenous Information State *958*
18.9 Closing Notes *959*
18.10 Bibliographic Notes *960*
Exercises *962*
Bibliography *967*

19 Direct Lookahead Policies *971*
19.1 Optimal Policies Using Lookahead Models *974*
19.2 Creating an Approximate Lookahead Model *978*
19.2.1 Modeling the Lookahead Model *979*
19.2.2 Strategies for Approximating the Lookahead Model *980*
19.3 Modified Objectives in Lookahead Models *985*
19.3.1 Managing Risk *985*
19.3.2 Utility Functions for Multiobjective Problems *991*
19.3.3 Model Discounting *992*
19.4 Evaluating DLA Policies *992*
19.4.1 Evaluating Policies in a Simulator *994*

19.4.2 Evaluating Risk-adjusted Policies *994*
19.4.3 Evaluating Policies in the Field *996*
19.4.4 Tuning Direct Lookahead Policies *997*
19.5 Why use a DLA? *997*
19.6 Deterministic Lookaheads *999*
19.6.1 A Deterministic Lookahead: Shortest Path Problems *1001*
19.6.2 Parameterized Lookaheads *1003*
19.7 A Tour of Stochastic Lookahead Policies *1005*
19.7.1 Lookahead PFAs *1005*
19.7.2 Lookahead CFAs *1007*
19.7.3 Lookahead VFAs for the Lookahead Model *1007*
19.7.4 Lookahead DLAs for the Lookahead Model *1008*
19.7.5 Discussion *1009*
19.8 Monte Carlo Tree Search for Discrete Decisions *1009*
19.8.1 Basic Idea *1010*
19.8.2 The Steps of MCTS *1010*
19.8.3 Discussion *1014*
19.8.4 Optimistic Monte Carlo Tree Search *1016*
19.9 Two-stage Stochastic Programming for Vector Decisions* *1018*
19.9.1 The Basic Two-stage Stochastic Program *1018*
19.9.2 Two-stage Approximation of a Sequential Problem *1020*
19.9.3 Discussion *1023*
19.10 Observations on DLA Policies *1024*
19.11 Bibliographic Notes *1025*
Exercises *1027*
Bibliography *1031*

Part VI – Multiagent Systems *1033*

20 Multiagent Modeling and Learning *1035*
20.1 Overview of Multiagent Systems *1036*
20.1.1 Dimensions of a Multiagent System *1036*
20.1.2 Communication *1038*
20.1.3 Modeling a Multiagent System *1040*
20.1.4 Controlling Architectures *1043*
20.2 A Learning Problem – Flu Mitigation *1044*
20.2.1 Model 1: A Static Model *1045*
20.2.2 Variations of Our Flu Model *1046*
20.2.3 Two-Agent Learning Models *1050*

20.2.4 Transition Functions for Two-Agent Model *1052*
20.2.5 Designing Policies for the Flu Problem *1054*
20.3 The POMDP Perspective* *1059*
20.4 The Two-Agent Newsvendor Problem *1062*
20.5 Multiple Independent Agents – An HVAC Controller Model *1067*
20.5.1 Model *1067*
20.5.2 Designing Policies *1069*
20.6 Cooperative Agents – A Spatially Distributed Blood Management Problem *1070*
20.7 Closing Notes *1074*
20.8 Why Does it Work? *1074*
20.8.1 Derivation of the POMDP Belief Transition Function *1074*
20.9 Bibliographic Notes *1076*
Exercises *1077*
Bibliography *1083*

Index *1085*

Preface

Preface to *Reinforcement Learning and Stochastic Optimization: A unified framework for sequential decisions*

This books represents a lifetime of research into what I now call sequential decision problems, which dates to 1982 when I was introduced to the problem arising in truckload trucking (think of Uber/Lyft for trucks) where we have to choose which driver to assign to a load, and which loads to accept to move, given the high level of randomness in future customer demands, representing requests to move full truckloads of freight.

It took me 20 years to figure out a practical algorithm to solve this problem, which led to my first book (in 2007) on approximate dynamic programming, where the major breakthrough was the introduction of the post-decision state and the use of hierarchical aggregation for approximating value functions to solve these high-dimensional problems. However, I would argue today that the most important chapter in the book (and I recognized it at the time), was chapter 5 on how to model these problems, without any reference to algorithms to solve the problem. I identified five elements to a sequential decision problem, leading up to the objective function which was written

$$\max_{\pi} \mathbb{E}\left\{\sum_{t=0}^{T} C(S_t, X^{\pi}(S_t))|S_0\right\}.$$

It was not until the second edition (in 2011) that I realized that approximate dynamic programming (specifically, policies that depend on value functions) was not the only way to solve these problems; rather, there were four classes of policies, and only one used value functions. The 2011 edition of the book listed three of the four classes of policies that are described in this book, but most of the book still focused on approximating value functions. It was not until a 2014

paper ("Clearing the Jungle of Stochastic Optimization") that I identified the four classes of policies I use now. Then, in 2016 I realized that the four classes of policies could be divided between two major strategies: the policy search strategy, where we search over a family of functions to find the one that works best, and the lookahead strategy, where we make good decisions by approximating the downstream impact of a decision made now.

Finally, I combined these ideas in a 2019 paper ("A Unified Framework for Stochastic Optimization" published in the *European Journal for Operational Research*) with a better appreciation of major classes of problems such as state-independent problems (the pure learning problems that include derivative-based and derivative-free stochastic search) and the more general state-dependent problems; cumulative and final reward objective functions; and the realization that any adaptive search algorithm was a sequential decision problem. The material in the 2019 paper is effectively the outline for this book.

This book builds on the 2011 edition of my approximate dynamic programming book, and includes a number of chapters (some heavily edited) from the ADP book. It would be nice to call this a third edition, but the entire framework of this book is completely different. "Approximate dynamic programming" is a term that still refers to making decisions based on the idea of approximating the downstream value of being in a state. After decades of working with this approach (which is still covered over a span of five chapters in this volume), I can now say with confidence that value function approximations, despite all the attention they have received, is a powerful methodology for a surprisingly narrow set of decision problems.

By contrast, I finally developed the confidence to claim that the four classes of policies are universal. This means that *any* method for making decisions will fall in one of these four classes, or a hybrid of two or more. This is a game changer, because it shifts the focus from an algorithm (the method for making decisions) to the model (specifically the optimization problem above, along with the state-transition function and the model of the exogenous information process). This means we write out the elements of a problem *before* we tackle the problem of designing policies to decisions. I call this:

Model first, then solve.

The communities working on sequential decision problems are very focused on methods, just as I was with my earlier work with approximate dynamic programming. The problem is that any particular method will be inherently limited to a narrow class of problems. In this book, I demonstrate how you can

take a simple inventory problem, and then tweak the data to make each of the four classes work best.

This new approach has opened up an entirely new way of approaching a problem class that, in the last year of writing the book, I started calling "sequential decision analytics," which is any problem consisting of the sequence:

Decision, information, decision, information, ...

I allow decisions to range from binary (selling an asset) to discrete choices (favored in computer science) to the high-dimensional resource allocation problems popular in operations research. This approach starts with a problem, shifts to the challenging task of modeling uncertainty, and then finishes with designing policies to make decisions to optimize some metric. The approach is practical, scalable, and universally applicable.

It is exciting to be able to create a single framework that spans 15 different communities, and which represents every possible method for solving sequential decision problems. While having a common language to model any sequential decision problem, combined with the general approach of the four classes of policies, is clearly of value, this framework has been developed by standing on the shoulders of the giants who have laid the foundational work for all of these methods. I have had to make choices regarding the best notation and modeling conventions, but my framework is completely inclusive of all the methods that have been developed to solve these problems. Rather than joining the chorus of researchers promoting specific algorithmic strategies (as I once did), my goal is to raise the visibility of all methods, so that someone looking to solve a real problem is working with the biggest possible toolbox, rather than just the tools developed within a specific community.

A word needs to be said about the title of the book. As this is being written, there is a massive surge of interest in "reinforcement learning," which started as a form of approximate dynamic programming (I used to refer to ADP and RL as similar to American English and British English). However, as the RL community has grown and started working on harder problems, they encountered the same experience that I and everyone else working in ADP found: value function approximations are not a panacea. Not only is it the case that they often do not work, they usually do not work. As a result, the RL community branched out (just as I did) into other methods such as "policy gradient methods" (my "policy function approximations" or PFA), upper confidence bounding (a form of "cost function approximation" or CFA), the original Q-learning (which produces a policy based on "value function approximations" or VFA), and finally

Monte Carlo tree search (a policy based on "direct lookahead approximations" or DLA). All of these methods are found in the second edition of Sutton and Barto's landmark book *Reinforcement Learning: An introduction*, but only as specific methods rather than general classes. This book takes the next step and identifies the general classes.

This evolution from one core method to all four classes of policies is being repeated among other fields that I came to call the "jungle of stochastic optimization." Stochastic search, simulation-optimization, and bandit problems all feature methods from each of the four classes of policies. Over time, I came to realize that all these fields (including reinforcement learning) were playing catchup to the grandfather of all of this work, which is optimal control (and stochastic control). The field of optimal control was the first to introduce and seriously explore the use of value function approximations (they call these cost-to-go functions), linear decision rules (a form of PFA), and the workhorse "model predictive control" (a great name for a simple rolling horizon procedure, which is a "direct lookahead approximation" in this book). I also found that my modeling framework was closest to that used in the optimal control literature, which was the first field to introduce the concept of a transition function, a powerful modeling device that has been largely overlooked by the other communities. I make a few small tweaks such as using state S_t instead of x_t, and decision x_t (widely used in the field of math programming) instead of u_t.

Then I introduce one big change, which is to maximize over all four classes of policies. Perhaps the most important innovation of this book is to break the almost automatic link between optimizing over policies, and then assuming that we are going to compute an optimal policy from either Bellman's equation or the Hamilton-Jacobi equations. These are rarely computable for real problems, which then leads people to assume that the natural next step is to approximate these equations. This is simply false, supported by decades of research where people have developed methods that do not depend on HJB equations. I recognize this body of research developing different classes of policies by making the inclusion of all four classes of policies fundamental to the original statement of the optimization problem above.

It will take some time for people from the different communities to learn to speak this common language. More likely, there will be an adaptation of existing modeling languages to this framework. For example, the optimal control community could keep their notation, but learn to write their objective functions as I have above, recognizing that the search over policies needs to span all four classes (which, I might point out, they are already using). I would hope that the reinforcement learning community, which adopted the notation for discrete action a, might learn to use the more general x (as the bandit community has already done).

I have tried to write this book to appeal to newcomers to the field, as well as people who already have training in one or more of the subfields that deal with decisions and uncertainty; recognizing these two broad communities was easily the biggest challenge while writing this book. Not surprisingly, the book is quite long. I have tried to make it more accessible to people who are new to the field by marking many sections with an * as an indication that this section can be skipped on a first-read. I also hope that the book will appeal to people from many application domains. However, the core audience is people who are looking to solve real problems by modeling applications and implementing the work in software. The notation is designed to facilitate writing computer programs, where there should be a direct relationship between the mathematical model and the software. This is particularly important when modeling the flow of information, something that is often overlooked in mainstream reinforcement learning papers.

Warren B. Powell

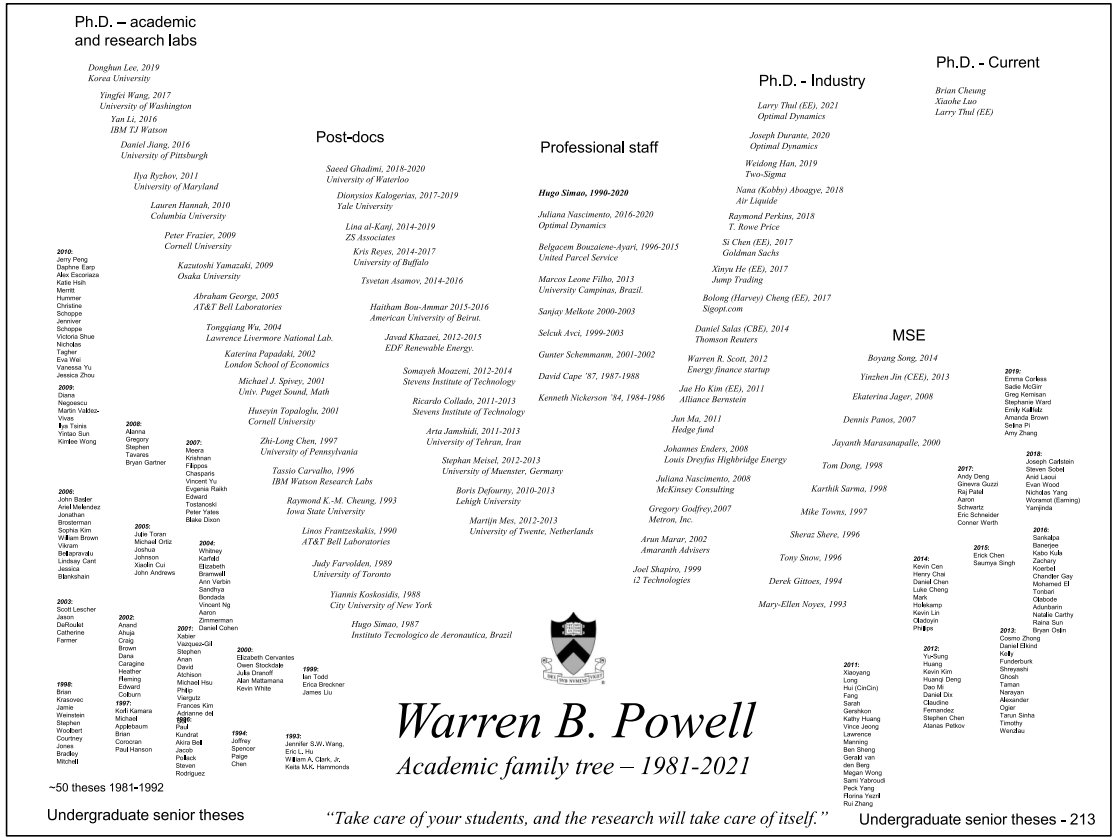
Princeton, New Jersey
August, 2021

Acknowledgments

The foundation of this book is a modeling framework for sequential decision problems that involves searching over four classes of policies for making decisions. The recognition that we needed all four classes of policies came from working on a wide range of problems spanning freight transportation (almost all modes), energy, health, e-commerce, finance, and even materials science (!!).

This research required a *lot* of computational work, which was only possible through the efforts of the many students and staff that worked in CASTLE Lab. Over my 39 years of teaching at Princeton, I benefited tremendously from the interactions with 70 graduate students and post-doctoral associates, along with nine professional staff. I am deeply indebted to the contributions of this exceptionally talented group of men and women who allowed me to participate in the challenges of getting computational methods to work on such a wide range of problems. It was precisely this diversity of problem settings that led me to appreciate the motivation for the different methods for solving problems. In the process, I met people from across the jungle, and learned to speak their language not just by reading papers, but by talking to them and, often, working on their problems.

I would also like to acknowledge what I learned from supervising over 200 senior theses. While not as advanced as the graduate research, the undergraduates helped expose me to an even wider range of problems, spanning topics such as sports, health, urban transportation, social networks, agriculture, pharmaceuticals, and even optimizing Greek cargo ships. It was the undergraduates who accelerated my move into energy in 2008, allowing me to experiment with modeling and solving a variety of problems spanning microgrids, solar arrays, energy storage, demand management, and storm response. This experience exposed me to new challenges, new methods, and most important, new communities in engineering and economics.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"Take care of your students, and the research will take care of itself."

The group of students and staff that participated in CASTLE Lab is much too large to list in this acknowledgment, but I have included my academic family tree above. To everyone in this list, my warmest thanks!

I owe a special thanks to the sponsors of CASTLE Lab, which included a number of government funding agencies including the National Science Foundation, the Air Force Office of Scientific Research, DARPA, the Department of Energy (through Columbia University and the University of Delaware), and Lawrence Livermore National Laboratory (my first energy sponsor). I would particularly like to highlight the Optimization and Discrete Mathematics Program of AFOSR that provided me with almost 30 years of unbroken funding. I would like to express my appreciation to the program managers of the ODM program, including Neal Glassman (who gave me my start in this program), Donald Hearn (who introduced me to the materials science program), Fariba Fahroo (whose passion for this work played a major role in its survival at AFOSR), and Warren Adams. Over the years I came to have a deep appreciation for the critical role played by these program managers who provide a critical bridge between academic researchers and the policymakers who have to then sell the work to Congress.

I want to recognize my industrial sponsors and the members of my professional staff that made this work possible. Easily one of the most visible features of CASTLE Lab was that we did not just write academic papers and run computer simulations; our work was implemented in the field. We would work with a company, identify a problem, build a model, and then see if it worked, and it often did not. This was true research, with a process that I once documented with a booklet called "From the Laboratory to the Field, and Back." It was this back and forth process that allowed me to learn how to model and solve real problems. We had some early successes, followed by a period of frustrating failures as we tackled even harder problems, but we had two amazing successes in the early 2000s with our implementation of a locomotive optimization system at Norfolk Southern Railway using approximate dynamic programming, and our strategic fleet simulator for Schneider National (one of the largest truckload carriers in the U.S.). This software was later licensed to Optimal Dynamics which is implementing the technology in the truckload industry. My industrial sponsors received no guarantees when they funded our research, and their (sometimes misplaced) confidence in me played a critical role in our learning process.

Working with industry from a university research lab, especially for a school like Princeton, introduces administrative challenges that few appreciate. Critical to my ability to work with industry was the willingness of a particular grants officer at Princeton, John Ritter, to negotiate contracts where companies funded the research, and were then given royalty-free licenses to use the

software. This was key, since it was through their use of the software that I learned what worked, and what did not. John understood that the first priority at a university is supporting the faculty and their research mission rather than maximizing royalties. I think that I can claim that my $50 million in research funding over my career paid off pretty well for Princeton.

Finally, I want to recognize the contributions of my professional staff who made these industrial projects possible. Most important is the very special role played by Hugo Simao, my first Ph.D. student who graduated, taught in Brazil, and returned in 1990 to help start CASTLE Lab. Hugo played so many roles, but most important as the lead developer on a number of major projects that anchored the lab, notably the multidecade relationship with Yellow Freight System/YRC. He was also the lead developer of our award-winning model for Schneider National that was later licensed to Optimal Dynamics, in addition to our big energy model, SMART-ISO, which simulated the PJM power grid. This is not work that can be done by graduate students, and Hugo brought his tremendous skill to the development of complex systems, starting in the 1990s when the tools were relatively primitive. Hugo also played an important role guiding students (graduate and undergraduate) with their software projects, given that I retired from programming in 1990 as the world transitioned from Fortan to C. Hugo brought talent, patience, and an unbelievable work ethic that provided the foundation in funding that made CASTLE Lab possible. Hugo was later joined by Belgacem Bouzaiene-Ayari who worked at the lab for almost 20 years and was the lead developer on another award-winning project with Norfolk Southern Railway, along with many other contributions. I cannot emphasize enough the value of the experience of working with these industrial sponsors, but this is not possible without talented research staff such as Hugo and Belgacem.

W. B. P.