On-line supplement to: SMART: A Stochastic Multi-scale Model for the Analysis of Energy Resources, Technology and Policy

This online supplement provides a more detailed version of the model, followed by derivations of the derivatives needed to fit the value function approximations.

A.1 The full energy model

We describe the various elements of the model using a notational style developed specifically for stochastic, dynamic resource allocation problems. We then model two types of decisions in our model: the dispatch decisions to supply energy to the various sources of demand which will be performed on an hourly basis, and the capacity acquisition decisions to add new capacity to the conversion plants (or to build new plants for a newly discovered technology) which will be done every year. We note that dispatch decisions include decisions of how much to use, how much to store, and how much to withdraw from storage. We let $\mathcal{T}$ denote the set of once-a-year time-periods where we add capacity and $\mathcal{H}$ the set of hourly periods within a year, where $|\mathcal{H}| = H$ and $|\mathcal{T}| = T$.

In this section, we provide the notation needed to model all the elements of a dynamic system, which is organized along five fundamental dimensions: 1) system state variables, 2) decision variables, 3) exogenous information (random variables), 4) transition function and 5) objective function. In section 5 of the paper, we use this modeling framework to formulate a deterministic linear programming model, and a stochastic optimization problem.

System state variables

We divide the state variable into the resource state (energy investments), storage (the amount of energy held in storage), and information about a range of system parameters.
Energy investments and storage
We let a represent a generic attribute vector. For example, the attribute vector of a conversion plant could be represented using

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \text{Type} \\ \text{Location} \\ \text{Age} \end{pmatrix}.$$

The attribute "Type" denotes the type of generation technology such as nuclear, gas turbine, photovoltaic or hydro-power. Age is typically in units of years. Location can be expressed at different levels of detail. For example, we may specify the location as a high-level region such as the central valley in California or at the more detailed level of a zip code. We conduct our numerical experiments only for a spatially aggregate model, but our mathematical model is much more general than this, and the algorithmic strategy scales fairly easily to more complex problems.

It is useful to define the following attribute vector spaces:

$$\begin{aligned}
\mathcal{A} &= \text{The set of all attribute vectors describing all possible investments in energy resources.} \\
\mathcal{A}^{Res} &= \text{The set of all attribute vectors describing resource nodes, where raw energy resources enter the network (coal, oil, wind, etc.).} \\
\mathcal{A}^{Conv} &= \text{The set of all attribute vectors describing conversion nodes, where raw resources are converted to electricity or other usable forms of energy.} \\
\mathcal{A}^{Stor} &= \text{The set of all attribute vectors describing storage nodes.} \\
\mathcal{A}^{Dem} &= \text{The set of all attribute vectors describing demand nodes.} \\
\mathcal{A}' &= \text{The set of all attribute vectors describing all other types of nodes, which include collection and distribution nodes used to simplify the linear program,} \\
&= \mathcal{A} \setminus (\mathcal{A}^{Res} \cup \mathcal{A}^{Conv} \cup \mathcal{A}^{Stor} \cup \mathcal{A}^{Dem}).
\end{aligned}$$

We define the connectivity of the energy network using

$$\begin{aligned}
\overrightarrow{\mathcal{A}}(a) &= \text{The set of attribute vectors of nodes downstream in the dispatch network that are directly connected to a node with attribute vector } a. \\
\overleftarrow{\mathcal{A}}(a) &= \text{The set of attribute vectors of nodes upstream in the dispatch network that are directly connected to a node with attribute vector } a.
\end{aligned}$$

Using these attributes, the resource state variables can be described using

$$\begin{aligned}
R_{tha} &= \text{The total capacity in megawatts (MW) with attribute vector } a \in \mathcal{A} \text{ available in hour } h \text{ of year } t. \\
R_{th} &= \text{The resource state vector in hour } h \text{ of year } t \\
&= (R_{tha})_{a \in \mathcal{A}}. \\
y_{tha} &= \text{The amount of energy in storage for a node with attribute vector } a \in \mathcal{A}^{Stor} \text{ in hour } h \text{ of year } t. \\
y_{th} &= (y_{tha})_{a \in \mathcal{A}^{Stor}}.
\end{aligned}$$

Exogenously varying quantities
In our model, energy demand is treated as exogenous, although we recognize that an important dimension of the energy problem involves demand management. We model demands similarly to how we model energy resources. We let $a \in \mathcal{A}^{Dem}$ be the attributes of demand, which will include the type of energy demand (electricity, natural gas, oil for heating, gasoline for transportation) and could also include location (in a model which captures where demand is occurring). This allows us to model demand as

$$\begin{aligned}
D_{tha} &= \text{Demand with attribute } a \in \mathcal{A}^{Dem} \text{ at hour } h \text{ in year } t. \\
D_{th} &= (D_{tha})_{a \in \mathcal{A}}.
\end{aligned}$$

Other exogenously varying parameters include

$$\begin{aligned} p_{th} &= \text{The amount of precipitation occurring in hour } h\text{, year } t. \\ \rho_{th} &= \text{The information state in hour } h \text{ of time } t\text{, which can include the state of technology } (\rho_{th}^{Tech})\text{, climate } (\rho_{th}^{Clim})\text{, market } (\rho_{th}^{Mkt}) \text{ and exogenous energy supply } (\rho_{th}^{exo}). \\ &= (\rho_{th}^{Tech}, \rho_{th}^{Clim}, \rho_{th}^{Mkt}, \rho_{th}^{exo}). \end{aligned}$$

The vector $\rho_{th}^{exo} = (\rho_{tha}^{exo})_{a \in \mathcal{A}^{Conv}}$ is used to model intermittent energy at conversion facilities. If the resource a is nonintermittent (coal, nuclear, natural gas), then $\rho_{tha}^{exo} = 1$. If it is an intermittent supply such as wind or solar, we would have $\rho_{tha}^{exo} < 1$ whenever the exogenous supply of energy does not allow us to use the technology (such as a wind turbine) to its fullest capacity.

We finally define the system state variable using

$$\begin{aligned} S_{th} &= \text{The state of the system in hour } h \text{ of year } t \\ &= (R_{th}, y_{th}, D_{th}, p_{th}, \rho_{th}). \end{aligned}$$

The system state could include additional variables. For example, if there is a constraint imposed on the total CO_2 emitted since the beginning of the time horizon, then the state could have the added dimension of the current level of CO_2 emissions.

Decision variables

Here we describe the different types of decisions and the constraints they have to observe at a point in time.

$$\begin{aligned} x_{tha}^{cap} &= \text{The total capacity with attribute vector } a \in \mathcal{A}^{Conv} \text{ added to the system in hour } h \text{ of year } t. \\ x_{th}^{cap} &= (x_{tha}^{cap})_{a \in \mathcal{A}^{Conv}}. \\ x_{th,aa'}^{disp} &= \text{The flow from a node with attribute vector } a \text{ to a node with attribute vector } a' \text{ during hour } h \text{ of year } t. \\ x_{th}^{disp} &= (x_{th,aa'}^{disp})_{a \in \mathcal{A}, a' \in \mathcal{A}}. \end{aligned}$$

The vector of decisions is given compactly by

$$\begin{aligned} x_{th} &= \text{The vector of decisions in hour } h \text{ of year } t \\ &= (x_{th}^{cap}, x_{th}^{disp}). \end{aligned}$$

The presentation of deterministic and stochastic optimization models which will determine how the decisions are made are given in section 5 of the main paper.

We note here that, although we have modeled all decisions at the hourly level, real-life capacity addition decisions are seldom made more than once a year. This does not require us to change any of the notation that has already been presented, but we assume that for all intermediate hours $0 < h \leq H$, $x_{th}^{cap} = 0$. On the other hand, the dispatch decisions for distributing power to the various sources of demand have to be made every hour $h \in \mathcal{H}$.

These decisions are governed by a series of constraints that apply to decisions made at a point in time. For all conversion nodes, the output is constrained by the installed capacity at that node, given by

$$\sum_{a' \in \overrightarrow{\mathcal{A}}(a)} x^{disp}_{th,aa'} \quad \leq \quad \rho^{exo}_{tha} R_{tha}, \quad a \in \mathcal{A}^{Conv}. \tag{14}$$

Here, R_{tha} gives the installed capacity, while ρ^{exo}_{tha} captures the effect of intermittent sources such as wind or solar.

Conversion nodes are the primary source of constraint in the supply of energy. In our numerical work, we assume that supplies at the resource nodes (which captures the inputs of coal, natural gas, and so on) are unbounded, but otherwise we can write

$$\sum_{a' \in \overrightarrow{\mathcal{A}}(a)} x^{disp}_{th,aa'} \leq R_{tha}, \quad a \in \mathcal{A}^{Res}. \tag{15}$$

Here, the units of R_{tha} can be in barrels of oil, cubic feet of natural gas or tons of biomass. Unit conversions are handled in the flow conservation constraints, which are given by

$$\sum_{a' \in \overleftarrow{\mathcal{A}}(a)} \theta_{a'a} x^{disp}_{th,a'a} - \sum_{a'' \in \overrightarrow{\mathcal{A}}(a)} x^{disp}_{th,aa''} \quad = \quad 0, \quad a \in \mathcal{A} \setminus \mathcal{A}^{Stor}, \tag{16}$$

$$y_{tha} + \sum_{a' \in \overleftarrow{\mathcal{A}}(a)} \theta_{a'a} x^{disp}_{th,a'a} - \sum_{a'' \in \overrightarrow{\mathcal{A}}(a)} x^{disp}_{th,aa''} \quad \geq \quad 0. \quad a \in \mathcal{A}^{Stor}. \tag{17}$$

Equation (17) captures our ability to draw down from the amount that we have stored. $\theta_{a'a}$ captures changes in units (barrels of oil, cubic feet of gas and gallons of water to megawatts) as well as losses due to transmission and storage. We note that the sum over $a'' \in \overrightarrow{\mathcal{A}}(a)$ includes flows from storage, and the sum over $a' \in \overleftarrow{\mathcal{A}}(a)$ includes flows that remain in storage. θ_{aa}, for $a \in \mathcal{A}^{stor}$, would capture losses of energy held in storage. We note that equations (16)-(17) represent constraints at a point in time. Later we present the equations that govern how storage levels evolve over time.

For all demand nodes, the total input should match the total demand,

$$\sum_{a' \in \overleftarrow{\mathcal{A}}(a)} x^{disp}_{th,a'a} \quad = \quad \hat{D}_{tha}, \quad \text{where } a \in \mathcal{A}^{Dem}, \tag{18}$$

where we assume that there is some expensive, exogenous source (such as imported electricity) to handle situations when all other sources are not sufficient to meet demand. Finally, we require

$$x^{disp}_{th} \geq 0. \tag{19}$$

Note that x^{cap}_{th} is unconstrained in sign, as negative capacity implies retirement. Let $\mathcal{X}^{disp}_{th}$ be the feasible region defined by equations (14) - (19).

Exogenous information

Exogenous information is expressed in the form of random variables governed by probability distributions or an exogenously provided file of scenarios. All random variables are denoted by "hats" (as in $\hat{D}$).

$\hat{R}_{th}$ = Exogenous changes in the resource state in hour h of year t (due, for example, to equipment failures or weather damage).

$\hat{D}_{tha}$ = Change in demand with attribute vector $a \in \mathcal{A}^{Dem}$ in hour h of year t.

$\hat{\rho}_{th}$ = Exogenous changes in any parameters governing technology, markets, climate and exogenous energy sources,
= $(\hat{\rho}_{th}^{Tech}, \hat{\rho}_{th}^{Mkt}, \hat{\rho}_{th}^{Clim}, \hat{\rho}_{th}^{exo})$.

$\hat{p}_{th}$ = The change in precipitation in hour h of year t.

We summarize all random variables in a single variable W_t (without a hat), as follows:

W_{th} = A vector of all exogenous information regarding parameters appearing in hour h of year t which can include data on weather, demands, technological parameters and market information,
= $\left(\hat{R}_{th}, \hat{D}_{th}, \hat{\rho}_{th}, \hat{p}_{th}\right)$.

The availability of technologies of each type would be an element of $\hat{\rho}_{th}^{Tech}$. Examples of components of $\hat{\rho}_{th}^{Clim}$ might be the level of allowable emissions out to year T, and the total quantities and hourly patterns of wind (which is a specific example of the exogenous production factor data, ρ_{tha}^{exo}). Interest rates, exogenous resource prices, and resource availability, would be included in $\hat{\rho}_{th}^{Mkt}$. Although all of these have been indexed at the hourly level, we note that each of these could be arriving on different time-scales. For example, information regarding climate data might be available to be used by the system at anywhere from an hourly to monthly frequency. Prices might change on a daily basis, while technological developments could take as long as a year to multiple years to impact the system.

Later, we need a more formal vocabulary to describe the stochastic process driving our system. Let Ω be the set of all possible realizations of the sequence $(W_{11}, \ldots, W_{1H}, W_{21}, \ldots, W_{2H}, \ldots, W_{TH})$ where T is the number of years and H is the number of time periods in a year. We let $W_{th}(\omega)$ be a sample realization of the random variables in W_{th} when we are following sample path ω. To complete our formalism, let $\mathcal{F}$ be the sigma-algebra on Ω, with filtrations $\mathcal{F}_{th} \subset \mathcal{F}_{t,h+1}$ and $\mathcal{F}_{tH} \subset \mathcal{F}_{t+1,0}$, where $\mathcal{F}_{th}$ is the sigma-algebra generated by the information up through year t, hour h. We use this notation below to describe our algorithm and establish properties of the decision function.

Transition function

The state of the system evolves as a result of the decisions taken as well as random events, which is written as,

$$\begin{aligned} S_{t,h+1} &= S^M\left(S_{th}, x_{th}, W_{t,h+1}\right), \quad h = 0, 1, \ldots, H-1, \\ S_{t+1,0} &= S^M\left(S_{tH}, x_{tH}, W_{t+1,0}\right). \end{aligned}$$

Elements of the system state may transition in different ways. For example, new capacity may be added to the system and old ones may be retired, giving us

$$R_{t,h+1,a} \quad = \quad R_{tha} + x^{cap}_{tha} + \hat{R}_{t,h+1,a}, \quad a \in \mathcal{A}^{Conv}, \tag{20}$$

where x^{cap}_{tha} is positive for capacity additions and negative when capacities are retired. $\hat{R}_{t,h+1,a}$ involves changes in the resource state as a result of unforeseen events such as Hurricane Katrina.

Changes in demand, precipitation and the parameter vector ρ_t are all handled similarly using

$$\hat{D}_{t,h+1} \quad = \quad D_{th} + \hat{D}_{t,h+1}, \tag{21}$$

$$\hat{p}_{t,h+1} \quad = \quad p_{th} + \hat{p}_{t,h+1}, \tag{22}$$

$$\rho_{t,h+1} \quad = \quad \rho_{th} + \hat{\rho}_{t,h+1}. \tag{23}$$

Let y_{tha} be energy stored in hour h of year t associated for storage location $a \in \mathcal{A}^{Stor}$. In our numerical work, we model the water reservoir as the only form of storage, but our model is much more general. We can represent the transition equation for storage facilities using

$$y_{t,h+1,a} \quad = \quad y_{tha} + \sum_{a' \in \overleftarrow{\mathcal{A}}(a)} \theta_{a'a} x^{disp}_{th,a'a} - \sum_{a'' \in \overrightarrow{\mathcal{A}}(a)} x^{disp}_{th,aa''} + \hat{p}_{t,h+1,a}, \quad a \in \mathcal{A}^{Stor} \tag{24}$$

where $\hat{p}_{t,h+1,a}$ captures exogenous precipitation into the reservoir with attribute a. A slightly modified equation is used when $h = H$. We note that the only flow between time periods occurs through storage nodes. All other flows of energy occur within a time period.

Cost functions

The costs consist of the costs for adding new capacity, the costs related to energy dispatch which cover the physical movement of energy resources and transmission of electrical energy, and costs for purchasing fuel from the various energy sources.

c^{cap}_{tha} = The capital cost for adding a unit of capacity with attribute vector $a \in \mathcal{A}^{Conv}$ in hour h of year t.

$c^{disp}_{th,aa'}$ = The cost of unit flow from node with attribute vector $a \in \mathcal{A}$ to node with attribute vector $a' \in \mathcal{A}$ in hour h of year t (for example, the transportation cost of moving biomass).

c^{f}_{tha} = Unit cost of fuel corresponding to resource node with attribute vector $a \in \mathcal{A}^{Res}$ in hour h of year t.

c^{p}_{tha} = Unit operating cost of a conversion plant with attribute vector $a \in \mathcal{A}^{Conv}$ in hour h of year t (assumed to be proportional to the flow).

Total capacity and dispatch costs are given by

$$\begin{aligned}
C_{th}^{cap}(S_{th}, x_{th}^{cap}) &= \text{The total capital costs in hour } h \text{ of year } t, \\
&= \sum_{a \in \mathcal{A}} c_{tha}^{cap} x_{tha}^{cap}. \\
C_{th}^{disp}(S_{th}, x_{th}^{disp}) &= \text{The total costs resulting from the dispatch of energy in hour } h \\
&\quad \text{of year } t, \\
&= \sum_{a \in \mathcal{A}} \sum_{a' \in \mathcal{A}} c_{th,aa'}^{disp} x_{th,aa'}^{disp} + \sum_{a \in \mathcal{A}^{Res}} \sum_{a' \in \mathcal{A}} c_{tha}^{f} x_{th,aa'}^{disp} + \sum_{a \in \mathcal{A}^{Conv}} \sum_{a' \in \mathcal{A}} c_{tha}^{p} x_{th,aa'}^{disp}.
\end{aligned}$$

We note that $x_{tha}^{cap} = 0$ for hours other than $h = 0$. c_{tha}^{cap} is the total purchase cost of the capacity being invested when $h = 0$. We may express the total cost function in hour h of year t using,

$$C_{th}(S_{th}, x_{th}) = C_{th}^{cap}(S_{th}, x_{th}^{cap}) + C_{th}^{disp}(S_{th}, x_{th}^{disp}).$$

A.2 Calculating the derivative $\hat{v}^{hydro}$

We begin by computing the marginal value of additional energy in a single, scalar storage which, for our numerical experiments, was represented as a water reservoir. We do this by computing the marginal impact of having an additional unit of water in storage at a point in time. This calculation is then used in the derivation of the derivative for additional capacity, which impacts an entire year.

The objective function at hour h is given by

$$\begin{aligned}
F_{th}^{\pi}(S_{th}) &= -C_{th}(S_{th}, X_{th}^{\pi}) - \sum_{h' > h} C_{th'}(S_{th'}, X_{th'}^{\pi}) - \sum_{t' > t} \sum_{h \in \mathcal{H}} C_{t'h}(S_{t'h}, X_{t'h}^{\pi}) && (25) \\
&= -C_{th}(S_{th}, X_{th}^{\pi}) + V_{th}^{x}(S_{th}^{x}), \\
&\approx -C_{th}^{disp}(S_{th}, X_{th}^{\pi}) - C_{th}^{cap}(S_{th}, X_{th}^{\pi}) + \bar{V}_{th}^{hydro}(y_{th}^{x}) + \bar{V}_{th}^{cap}(R_{th}^{x}). && (26)
\end{aligned}$$

Note that $C_{th}^{cap}(S_{th}, X_{th}^{\pi}) = 0$ for all $h > 0$ since we assume that capacity addition decisions are made only at the beginning of each year.

To update $\left\{\bar{V}_{th}^{hydro}(y_{th}^{x})\right\}_{h \in \mathcal{H}, t \in \mathcal{T}}$, we need to differentiate $F_{th}^{\pi}(S_{th})$ with respect to y_{th} which is given by

$$\hat{v}_{th}^{hydro} = \frac{\partial F_{th}^{\pi}(S_{th})}{\partial y_{th}}.$$

We note that we differentiate around the pre-decision state y_{th}, but then use this to update $\bar{V}_{t,h-1}^{x}(y_{t,h-1}^{x})$ around the previous post-decision state $y_{t,h-1}^{x}$. We may reasonably claim that

$$\frac{\partial R_{th}^{x}}{\partial y_{th}} \approx 0 \qquad (27)$$

which captures the belief that an incremental change in the amount of energy in storage will have virtually no impact on our investments in energy resources. For example, an incremental increase in the amount stored in a reservoir will not have a measurable impact on the number of wind turbines we would purchase next year. This approximation gives us

$$
\begin{aligned}
\hat{v}^{hydro}_{th} &= \frac{\partial F^{\pi}_{th}(S_{th})}{\partial y_{th}} && (28)\\
&\approx \frac{\partial}{\partial y_{th}}\left(-C^{disp}_{th}(S_{th}, X^{\pi}_{th}) + \bar{V}^{hydro}_{th}(y^{x}_{th}) + \bar{V}^{cap}_{th}(R^{x}_{th})\right)\\
&= \frac{\partial}{\partial y_{th}}\left(-C^{disp}_{th}(S_{th}, X^{\pi}_{th}) + \bar{V}^{hydro}_{th}(y^{x}_{th})\right) + \frac{\partial \bar{V}^{cap}_{th}(R^{x}_{th})}{\partial R^{x}_{th}}\frac{\partial R^{x}_{th}}{\partial y_{th}}\\
&\approx \frac{\partial}{\partial y_{th}}\left(-C^{disp}_{th}(S_{th}, X^{\pi}_{th}) + \bar{V}^{hydro}_{th}(y^{x}_{th})\right) && (29)
\end{aligned}
$$

where we use the approximation, $\partial R^{x}_{th}/\partial y_{th} \approx 0$. The derivative (29) is approximated using a finite difference. Let S^{+}_{th} be the resource state where y_{th} is incremented by 1.

We then approximate the derivative using

$$
\hat{v}^{hydro}_{th} \approx F^{disp,\pi}_{th}(S^{+}_{th}) - F^{disp,\pi}_{th}(S_{th}),
$$

where $F^{disp,\pi}_{th}(S_{th}) = -C^{disp}_{th}(S_{th}, X^{\pi}_{th}) + \bar{V}^{hydro}_{th}(y^{x}_{th})$, denotes the dispatch optimization component of the objective function. This numerical derivative is exceptionally fast to compute since it involves a trivial reoptimization of a small linear program.

It is important to emphasize that the calculation of $\hat{v}^{hydro}_{th}$ captures not only the hourly variability of wind, but also the uncertainty, since the value of additional stored energy does not require knowing the future about wind. The marginal value is also computed in the context of a storage facility.

A.3 Calculating the derivative $\hat{v}^{cap}$

The procedure for updating $\left\{\bar{V}^{cap}_{th}(R^{x}_{th})\right\}_{h\in\mathcal{H},t\in\mathcal{T}}$ is somewhat more involved. We assume that investment decisions are made once each year, taking effect in a future year. Our goal is to calculate the marginal value of an additional unit of energy conversion capacity from a decision made in a previous year.

Changing the resource vector at the beginning of the year changes the resource vector for the entire year through equation (14). Also, since we only make capacity decisions once each year, we do not compute $\bar{V}^{cap}_{th}(R^{x}_{th})$ for all h, but rather only for $h = 0$. For this reason, we have to capture the marginal value of additional capacity on both the yearly investment problem (equation (11) of the main paper) as well as all the hourly dispatch problems.

We start with F^{π}_{t0} given by equation (25), although we will use the approximation in (26). Our goal is to compute

$$
\hat{v}^{cap}_{t0a} = \frac{\partial F^{\pi}_{t0}(S_{t0})}{\partial R_{t0a}}. \tag{30}
$$

F^{π}_{t0} can be written as a function of $R_{t0}, \ldots, R_{th}, \ldots, R_{t,H-1}$, recognizing that $R_{th} = R_{t0}$ for $0 \leq h < H$. R_{th} impacts the dispatch decision at hour h, as well as the post-decision storage level y^x_{th} which impacts future problems. In addition, changing R_{t0} has an impact on $R_{t+1,0}$, which impacts decisions in future years. To capture these interactions, we write

$$\frac{dF^\pi_{t0}(S_{t0})}{dR_{t0a}} = \left(-\frac{\partial C^{cap}_{t0}(S_{t0}, X^\pi_{t0})}{\partial R_{t0a}} + \frac{\partial F^\pi_{t+1,0}}{\partial R_{t+1,0}}\frac{\partial R_{t+1,0}}{\partial R_{t0}}\right) - \sum_{0\leq h<H}\frac{\partial F^\pi_{t0}}{\partial R_{th}}\frac{\partial R_{th}}{\partial R_{t0}}. \tag{31}$$

We can compute the marginal value of additional resources for the capacity subproblem at hour $h = 0$, given by equation (11) of the main paper, by using either the dual variable for equation (12) in the main paper, or a finite difference. We redefine S^+_{t0} here to be S_{t0} with R_{t0} incremented by 1. The finite difference is then given by

$$\hat{v}^+_{t0a} = F^{cap,\pi}_{t0}(S^+_{t0}) - F^{cap,\pi}_{t0}(S_{t0}).$$

This approximates the two terms in the set of parentheses on the right hand side of (31).

We next observe that

$$\frac{\partial F^\pi_{t0}}{\partial R_{th}} = \frac{\partial F^\pi_{th}}{\partial R_{th}}$$

since changing R_{th} has no impact on the problem for earlier hours, and

$$\frac{\partial R_{th}}{\partial R_{t0}} = 1.$$

The derivative $\partial F^\pi_{th}/\partial R_{th}$ involves finding the derivative of $F^\pi_{th}(S_{th})$ approximated by (26), which we compute using the dual variable for equation (14). Let ν^{disp}_{tha} be the dual variable for (14). Since the right hand side of (14) is $\rho^{exo}_{tha}R_{tha}$, the marginal impact of increasing R_{tha} on the dispatch optimization component of the objective function is given by

$$\frac{\partial F^\pi_{th}(S_{th})}{\partial R_{tha}} = \frac{\nu^{disp}_{tha}}{\rho^{exo}_{tha}}.$$

We finally pull these calculations together to find the marginal value of an additional unit of energy resource at the beginning of the year using

$$\hat{v}^{cap}_{t0a} \approx \sum_{h\in\mathcal{H}}\frac{\nu^{disp}_{tha}}{\rho^{exo}_{tha}} + \hat{v}^+_{t0a}. \tag{32}$$

We use $\hat{v}^{cap}_{t0a}$ to update the value function approximation $\bar{V}^{cap}_{t-1,0a}$.

A.4 The complete ADP algorithm

We close our presentation with a complete summary of the ADP algorithm, given in figure 7.

Step 0: Set the iteration counter, $n = 1$. Initialize the value function approximations $\{\bar{V}_{t0}^{cap,0}(R_{t0}^x) = 0\}_{t\in\mathcal{T}}$ and $\{\bar{V}_{th}^{hydro,0}(y_{th}^x)\}_{h\in\mathcal{H},t\in\mathcal{T}}$. Initialize the state S_{0a}^0. Set $t = 0$.

Step 1: Solve the annual problem of capacity addition:

$$F_{t0}^{cap}(S_{t0}^n) \quad = \quad \max_{x_{t0}^{cap}\geq 0} \left(- C^{cap}\left(S_{t0}^n, x_{t0}^{cap}\right) + \bar{V}_{t0}^{cap,n-1}(R_{t0}^x)\right) \tag{33}$$

We let $x_{t0}^{cap,n}$ denote the argument that solves equation (33) and $\hat{v}_{t0}^{+,n} = (\hat{v}_{t0a}^{+,n})_{a\in\mathcal{A}}$, the vector of numerical derivatives where $\hat{v}_{t0a}^{+,n} = [F_{t0}^{cap}(S_{t0}^n, R_{t0a}^n + 1) - F_{t0}^{cap}(S_{t0}^n, R_{t0a}^n] \quad \forall\, a \in \mathcal{A}$.

Step 2: Set the hour, $h = 0$.

Step 2a: Observe $W_t(\omega^n) = (\hat{R}_t, \hat{D}_t, \hat{\rho}_t, \hat{p}_t)$.

Step 2b: Solve the hourly optimization model (equation (10)) to obtain $F_{th}^{disp}(S_{th}^n)$. Let $\nu_{th}^{disp,n}(= (\nu_{tha}^{disp,n})_{a\in\mathcal{A}})$ be the vector of dual values corresponding to the flow conservation constraints represented by equation (3) and let $\hat{v}_{th}^{hydro,n}$ be the numerical derivative $\left[F_{th}^{disp}(S_{th}^n, y_{th}^n + 1) - F_{th}^{disp}(S_{th}^n, y_{th}^n)\right]$.

Step 2c: Sample the incoming precipitation data, $\hat{p}_{t,h+1}$ in order to compute the new reservoir level using equation (11).

Step 2d: Set $h = h + 1$. If $h < H$, go to **step 2**. Otherwise, continue.

Step 2e: Update the value function approximations for hydro-storage,

$$\bar{V}_{t,h-1}^{hydro,n}(y_{t,h-1}^x) \quad \leftarrow \quad U^V\left(\bar{V}_{t,h-1}^{hydro,n-1}, y_{t,h-1}^x, \hat{v}_{th}^{hydro,n}\right) \quad \forall\, h.$$

Step 2f: Compute the hourly state transitions:

$$S_{t,h+1}^n \quad = \quad S^M\left(S_{th}^n, x_{th}^n, W_{t,h+1}^n\right)$$

Step 3: Compute the combined dual values (see (32) in the online supplement).

$$\hat{v}_{ta}^{cap,n} \quad = \quad \hat{v}_{t0a}^{+,n} + \sum_{h\in\mathcal{H}} \frac{\nu_{tha}^{disp,n}}{\hat{\rho}_{tha}^{exo}} \quad \forall\, a \in \mathcal{A}.$$

Step 4: Update the value function approximations for the capacity acquisitions,

$$\bar{V}_{t-1,0}^{cap,n}(R_{t-1,0}^x) \quad \leftarrow \quad U^V\left(\bar{V}_{t-1,0}^{cap,n-1}, R_{t-1,0}^{x,n}, \hat{v}_t^{cap,n}\right).$$

Step 5: Compute the state transitions:

$$\begin{aligned} S_{t,h+1}^n \quad &= \quad S^M\left(S_{th}^n, x_{th}^n, W_{t,h+1}^n\right) \\ S_{t+1,0}^n \quad &= \quad S^M\left(S_{tH}^n, x_{tH}^n, W_{t+1,0}^n\right). \end{aligned}$$

Step 6: Set $t = t + 1$. If $t < T$, go to **step 1**. Otherwise, continue.

Step 7: Set $n = n + 1$. If $n < N$, set $t = 0$ and go to **step 1**. Otherwise, return the value functions $\left\{\bar{V}_{t0}^{cap,n}(R_{t0}^x)\right\}_{t\in\mathcal{T}}$ and $\left\{\bar{V}_{th}^{hydro,n}(y_{th}^x)\right\}_{h\in\mathcal{H},t\in\mathcal{T}}$, which define the policy for adding capacity over the years $t \in \mathcal{T}$.

Figure 7: The ADP algorithm to solve the energy resource management problem.